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Grasping Force Optimization Algorithm of Soft Multi-fingered Hand Based on Safety Margin Detection

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Abstract: The classical gradient flow optimization algorithm requires a valid initial point before starting the recursive algorithm, and the existing methods can't guarantee that the initial values fully satisfy the friction cone constraints of contact point in the optimization process of gradient flow algorithm. In order to improve safety margin and prevent the finger from slipping at contact point, we present an iterative method of safe initial values with safety margin detection and develop a gradient flow optimization algorithm based on the safe initial values. Firstly, the safety margin is defined which more intuitively reflects the margin of the grasping forces at contact point. The resulting safe initial values can be achieved by the detection of desired safety margin at each iteration. Secondly, the safe initial values are usually not optimal, even with the valid initial values, and it can't always ensure that the finger contact force always satisfies the friction cone constraints during the optimization. It is an effective way to eliminate the unreliable initial values in the optimization and obtain a safer initial values by increasing the safety margin. By transforming the safe initial values into an initial point of the gradient flow algorithm, the final optimized values of grasping forces can be generated efficiently by gradient flow iteration. Grasp examples of the soft multi-fingered hand indicate the effectiveness of the general solution of the force optimization algorithm based on safety margin detection. The method eliminates the shortcomings of the gradient flow optimization process caused by the initial value problem and provides a more accurate and reliable force optimization result for multi-fingered dexterous manipulation.

Keywords: force optimization; grasping force; multi-fingered hand; soft finger; gradient flow; friction cone

1 Introduction

Multi-fingered hands have attracted much attention in robot manipulation with the dexterity to secure target objects of different sizes, shapes, and orientations[1-3]. Force optimization is a vital problem in dexterous manipulation, which determines the optimal grasping forces to balance external wrench acting on the grasped object and maintain grasp stability[4].

Many algorithms have been proposed for the force optimization problem of multi-fingered hand grasp. Therefore, early works linearize the friction cone with the polyhedral cone and the linear programming method to solve optimal forces[5-6]. Recently, Zheng et al. developed an optimization algorithm to compute a set of primitive forces and the minimum forces faster than the earlier computation methods[7]. Most of them involve a large number of calculations, and the resulting grasping forces are usually conservative.

A classical linearly constrained gradient flow optimization algorithm was presented by Buss[8]. The force optimization problem has been formulated as a convex optimization problem on a Riemannian manifold with linear constraints. On this basis, some algorithms have been further explored and developed[4, 9-15]. An improved recursion was the Dikin-type algorithm for the general class of strictly convex twice-continuously differentiable cost functions presented[9]. To reduce computational complexity, the computation of the solution was split into online and offline components by using sparse matrix techniques and reducing matrix dimension[10-11]. A general force optimization of multi-fingered grasp with hard-finger contact was proposed based on Lagrange initial values[12]. The gradient flow optimization algorithm was also extended to the multi-arm robots with multi-fingered hands[13-14]. The above algorithms don't linearize the friction cone, thus improving the accuracy of force optimization, and these

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results of nonlinear methods are more reasonable.

But these algorithms based on gradient flow also have a shortcoming, that is, the valid initial values of grasping forces are provided before starting iteration. For this reason, the Max-Det algorithm using linear matrix inequality was presented for initial grasping forces, however, this method suffered from singularity issues in some cases[15]. The initial values can be computed with the proposed methods but at the expense of a significant computational effort[16]. A simple single-valued optimization problem was constructed to obtain the initial values[17]. An iterative method of initial values was presented based on the Lagrange multipliers[12], and the initial values that satisfy frictional constrains could be obtained autonomously by adjusting the weight factors of the normal forces. Recently, a linear combination method of the grasping forces was demonstrated for initial values[18]. Due to the complexity of grasp, the existing methods of initial values can't guarantee that the contact force fully satisfies the friction cone constraints during the optimization of gradient flow algorithm, and there is no way to provide a remedy.

To get safer initial values and more intuitively reflect the margin of grasping forces, we present an iterative method of the initial values of grasping forces with safety margin detection. It can always ensure that the safe initial values satisfy the friction cone constraints during the optimization of the gradient flow algorithm.

2 Safety margin definition and force closure grasping

2.1 Safety margin definition

The contacts between the fingers and an object are divided into frictionless point contact, point contact with friction (hard finger contact) and soft finger contact. The contact force of soft finger is usually described by four force components at contact point i, that is $\boldsymbol{f}_{ci} = \left[f_{ci}^1, f_{ci}^2, f_{ci}^3, f_{ci}^4\right]^{\mathrm{T}} \in \mathrm{R}^4$. Where f_{ci}^1 and f_{ci}^2 are tangential force components, $f_{ci}^3 > 0$ and f_{ci}^4 are the normal force and torsion components around the contact normal on the object surface. An elliptical approximation model is usually adopted to describe the friction cone constraints of soft finger contact:

$$\frac{(f_{ci}^1)^2 + (f_{ci}^2)^2}{u_i} + \frac{(f_{ci}^4)^2}{u_{t,i}} \leqslant (f_{ci}^3)^2 \tag{1}$$

Where u_i and $\mu_{t,i}$ are the friction coefficient and torsional coefficient at contact point.

The nonlinear friction cones and unidirectional normal force constraints are described as a specific structure with linear constraints of symmetric positive definite matrix[8-9]. The commonly elliptical approximation friction model of soft finger at the i-th contact point can be described as:

$$\boldsymbol{P}_i = \begin{bmatrix} f_{ci}^3 & 0 & 0 & \alpha_i f_{ci}^1 \\ 0 & f_{ci}^3 & 0 & \alpha_i f_{ci}^2 \\ 0 & 0 & f_{ci}^3 & \beta_i f_{ci}^4 \\ \alpha_i f_{ci}^1 & \alpha_i f_{ci}^2 & \beta_i f_{ci}^4 & f_{ci}^3 \end{bmatrix} \in \mathrm{R}^{4\times 4} \tag{2}$$

Where $\alpha_i = \sqrt{1/u_i}$, $\beta_i = \sqrt{1/u_{t,i}}$.

A special eigenvalue of the positive matrix is:

$$\lambda_{ci} = f_{ci}^3 - \sqrt{1/u_i((f_{ci}^1)^2 + (f_{ci}^2)^2) + 1/u_{t,i}(f_{ci}^4)^2} \tag{3}$$

Where $\lambda_{ci} > 0$, and $f_{ci}^3 > 0$ denotes the grasping forces satisfying friction cone constraints at contact point. In order to avoid slip of soft finger at contact point, a safety margin is defined, which more intuitively reflects the margin of the grasping forces at contact point i:

$$\rho_i = \lambda_{ci}/f_{ci}^3 \in [0,1], \quad i = 1,2,\cdots,k \tag{4}$$

Where ρ_i represents the contact safety margin with a range of 0～100% at contact point in theory. By automatically increasing the normal component of the internal force, the change in each component of the grasping force is evaluated to achieve the desired safety margin. Minimum safety margin ($\rho_i \to 0$), likely causes the grasping forces to approach the edge of friction cone and raises the possibility of slip at the contact point. When the safety margin increases, the grasping forces are away from the edge of the friction cone, which improves the stability at the contact point. Therefore, it is necessary to increase suitable safety margin at contact point for multi-fingered hand grasp.

2.2 Force closure grasping

The grasping forces exerted by k soft fingers can be represented as $\boldsymbol{f}_{\mathrm{c}} = \left[\boldsymbol{f}_{c1}^{\mathrm{T}}, \cdots, \boldsymbol{f}_{ci}^{\mathrm{T}}, \cdots, \boldsymbol{f}_{ck}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathrm{R}^{4k}$. Each grasping forces exerted by a single contact can be written as a grasp matrix $\boldsymbol{G}_i \in \mathrm{R}^{6\times 4}$.

$$\boldsymbol{G}_i = \begin{bmatrix} \boldsymbol{t}_i & \boldsymbol{\tau}_i & \boldsymbol{n}_i & 0 \\ \boldsymbol{r}_i \times \boldsymbol{t}_i & \boldsymbol{r}_i \times \boldsymbol{\tau}_i & \boldsymbol{r}_i \times \boldsymbol{n}_i & \boldsymbol{n}_i \end{bmatrix} \tag{5}$$

Where $\boldsymbol{t}_i$, $\boldsymbol{\tau}_i$, $\boldsymbol{n}_i$ and $\boldsymbol{r}_i$ are two unit orthogonal tangent vectors, a unit inward normal vector and a position vector at contact point i written in the object coordinate frame. The grasping forces of each finger at contact point along unit vectors $\boldsymbol{t}_i$, $\boldsymbol{\tau}_i$ and $\boldsymbol{n}_i$ can be represented as $\boldsymbol{f}_{ci} = \left[f_{ci}^1, f_{ci}^2, f_{ci}^3, f_{ci}^4\right]^{\mathrm{T}} \in \mathrm{R}^4$. In order to determine the effects of grasping forces, all the forces can be transformed to the object coordinate frame. The grasp is a force closure grasp if given any external wrench $\boldsymbol{f}_e \in \mathrm{R}^6$ applied to the object, there exist contact forces such that:

$$\boldsymbol{G}\boldsymbol{f}_c + \boldsymbol{f}_e = 0 \tag{6}$$

Where $\boldsymbol{G} = [\boldsymbol{G}_1, \cdots, \boldsymbol{G}_k] \in \mathrm{R}^{6\times 4k}$ and $\boldsymbol{G}\boldsymbol{f}_c \in \mathrm{R}^6$ represent the total grasping matrix and the object wrench respectively.

3 Safe initial values of grasping forces with safety margin detection

If the grasp matrix $\boldsymbol{G}$ satisfies $\boldsymbol{f}_e \in R(\boldsymbol{G})$, the general solution of grasping forces $\boldsymbol{f}_c \in \mathrm{R}^{4k}$ can be expressed as particular solution and homogeneous solution to Eq.(6), namely:

$$\boldsymbol{f}_c = \boldsymbol{f}_{ext} + \boldsymbol{f}_{int} = -\boldsymbol{G}^+\boldsymbol{f}_e + (\boldsymbol{I} - \boldsymbol{G}^+\boldsymbol{G})\boldsymbol{\delta} \tag{7}$$

To expand the formula, each component of the grasping forces can be written as:

$$\begin{bmatrix} \boldsymbol{f}_{c1} \\ \vdots \\ \boldsymbol{f}_{ci} \\ \vdots \end{bmatrix} = \left(\boldsymbol{G}^+\boldsymbol{f}_e\right) + (\boldsymbol{I} - \boldsymbol{G}^+\boldsymbol{G}) \begin{bmatrix} \boldsymbol{\delta}_1 \\ \vdots \\ \boldsymbol{\delta}_i \\ \vdots \end{bmatrix} \tag{8}$$

$$\boldsymbol{\delta} = \left[\boldsymbol{\delta}_1^{\mathrm{T}}, \cdots, \boldsymbol{\delta}_i^{\mathrm{T}}, \cdots, \boldsymbol{\delta}_k^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathrm{R}^{4k} \tag{9}$$

Where $\boldsymbol{G}^+ = \boldsymbol{G}^{\mathrm{T}}(\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}})^{-1}$ is a pseudo-inverse matrix of the grasping matrix $\boldsymbol{G}$, $\boldsymbol{I}$ is the identity matrix, and $\boldsymbol{\delta} \in \mathrm{R}^{4k}$ is a fixed weight coefficient column vector. By the Eq.(7), the grasping forces consist of operating forces $\boldsymbol{f}_{ext}$ and internal forces $\boldsymbol{f}_{int}$. The operating forces, namely the particular solution of Eq.(7), can be used to balance gravity force and inertial effects to control the object. The internal forces, namely the homogeneous solution, are contact wrenches within the null space that satisfy the friction constraints and don't contribute to the object motion.

Indeed, it doesn't ensure that this particular solution always satisfies the nonlinear friction cones before the algorithm starting iteration by Eq.(7). A practical discrete iterative algorithm is necessary to adjust the weight parameters of internal forces to satisfy friction cone constraints now. Firstly, initialize each component of the grasping forces by k contact points of soft fingers.

$$\boldsymbol{\delta}(0) = \left[\boldsymbol{\delta}_1^{\mathrm{T}}, \cdots, \boldsymbol{\delta}_i^{\mathrm{T}}, \cdots, \boldsymbol{\delta}_k^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathrm{R}^{4k} \tag{10}$$

$$\boldsymbol{\delta}_i = [0, 0, \varepsilon\delta_0, 0]^{\mathrm{T}} \in \mathrm{R}^4 \tag{11}$$

Where $\boldsymbol{\delta}(0)$ is the initial step length value corresponding to internal forces in contact normal direction. The values of the constants $\delta_0 > 0$ and $\varepsilon > 1$ are two fixed positive real numbers to ensure that the constant component $\varepsilon\delta_0$ is always greater than δ_0. The normal component of the internal forces multiplies by the constant $\varepsilon\delta_0$ to make the internal forces increase at each iteration continuously.

Secondly, start the following discrete iterative equation.

$$\boldsymbol{f}_c(j) = \boldsymbol{G}^+\boldsymbol{f}_e + (\boldsymbol{I} - \boldsymbol{G}^+\boldsymbol{G})\boldsymbol{\delta}(j) \tag{12}$$

Where $\boldsymbol{G}^+\boldsymbol{f}_e$ and $\boldsymbol{I} - \boldsymbol{G}^+\boldsymbol{G}$ are constant matrix and don't change in iteration. The step length $\boldsymbol{\delta}(j)$ can be increased in each iteration:

$$\boldsymbol{\delta}(j) = \varepsilon^j\boldsymbol{\delta}(0), \quad j = 1, 2, \cdots \tag{13}$$

Where the step length $\boldsymbol{\delta}(j)$ at j loop iteration is ε^j times compared with the initial step length value $\boldsymbol{\delta}(0)$. By Eq.(12) and (13), the general discrete iteration formula of grasping forces can be rewritten as:

$$\boldsymbol{f}_{ci}(j) = \boldsymbol{G}^+\boldsymbol{f}_e + (\boldsymbol{I} - \boldsymbol{G}^+\boldsymbol{G})\varepsilon^j\boldsymbol{\delta}(0) \tag{14}$$

The discrete contact safety margin ρ_i for the i-th contact point at the j-th iteration can be written as:

$$\rho_i(j) \geqslant \lambda_{ci}(j)/f_{ci}^3(j), \ \ i = 1, 2, \cdots; \ j = 1, 2, \cdots \tag{15}$$

Where $\rho_i(j)$ denotes the safety margin at contact point i in j loop iteration. When the safety margin $\rho_i(j)$ is close to 0, the grasping forces are close to the edge of the friction cone, which raises the possibility of slip at the contact point. When the safety margin $\rho_i(j)$ increases, the grasping forces are away from the edge of the friction cone, which increases the stability at the contact point. Expand Eq.(15), yielding:

$$\boldsymbol{\rho}(j) = [\rho_1(j), \rho_2(j), \cdots, \rho_k(j)]^{\mathrm{T}} \tag{16}$$

With the help of safety detector, it can ensure that the safe initial values of grasping forces always satisfy the friction cone constraints and force closure during the optimization of the gradient flow algorithm in each iteration.

4 Linearly constrained gradient flow optimization algorithm

The classical gradient flow optimization algorithm was proposed by Buss[8-9]. Consider a k soft-fingered hand grasping an object, all the friction cone constraints can be expressed as a block diagonal matrix.

$$\boldsymbol{P} = \text{blockdiag}(\boldsymbol{P}_1, \boldsymbol{P}_2, \cdots, \boldsymbol{P}_k) \tag{17}$$

Where the friction matrix $\boldsymbol{P}_1 \in \mathrm{R}^{4\times4}$, $\boldsymbol{P}_2 \in \mathrm{R}^{4\times4}$ and $\boldsymbol{P}_k \in \mathrm{R}^{4\times4}$ are block diagonal matrices, and the positive definiteness of the linearly constrained matrix $\boldsymbol{P}$ is a necessary and sufficient condition for $\boldsymbol{P}$ to satisfy friction force limit constraints ($\boldsymbol{P} \in \mathrm{R}^{4k\times4k}$). It is described that the special structure matrix can be converted to linear homogeneous equation, and the form of such linear constraints can be written as:

$$\boldsymbol{A}_1 \text{vec}(\boldsymbol{P}) = 0 \tag{18}$$

Where $\boldsymbol{A}_1 \in \mathrm{R}^{km\times l}$ is a matrix with $l = 64k$, m is the number of linear equality constraints with rank of m and $\text{vec}(\boldsymbol{P}) \in \mathrm{R}^{(4k)^2}$ is a vector quantization operator. Also, in order to balance external force on the object, the force closure grasping can be rearranged into a linear equation:

$$\boldsymbol{A}_2 \text{vec}(\boldsymbol{P}) = -\boldsymbol{f}_\mathrm{e} \tag{19}$$

Where $\boldsymbol{A}_2 \in \mathrm{R}^{6\times(4k)^2}$ is determined by a linear map between $\boldsymbol{G}$, $\boldsymbol{P}$ and $\boldsymbol{f}_\mathrm{e}$. By incorporating Eq.(18) and (19), the general form of such linear constraints of $\boldsymbol{P}$ can be expressed as:

$$\boldsymbol{A} \text{vec}(\boldsymbol{P}) = \boldsymbol{q} \tag{20}$$

Where all the friction cone constraints and force closure grasping are transformed into a simple matrix equation, $\boldsymbol{A} = [\boldsymbol{A}_1, \boldsymbol{A}_2]^\mathrm{T} \in \mathrm{R}^{(mk+6)\times(4k)^2}$, $\boldsymbol{q} = [0, -\boldsymbol{f}_\mathrm{e}]^\mathrm{T} \in \mathrm{R}^{mk+6}$.

The classic linearly constrained gradient flow approach is based on the minimization of the cost function $\psi(\boldsymbol{P})$:

$$\min \psi(\boldsymbol{P}) = \text{tr}(\boldsymbol{W}_p \boldsymbol{P} + \boldsymbol{W}_i \boldsymbol{P}^{-1}) \quad \text{s.t.} \ \boldsymbol{A} \text{vec}(\boldsymbol{P}) = \boldsymbol{q} \tag{21}$$

Where tr() denotes a trace operator of matrix. Both fixed weight $\boldsymbol{W}_p \in \mathrm{R}^{4k\times4k}$ and $\boldsymbol{W}_i \in \mathrm{R}^{4k\times4k}$ are symmetric positive definite matrices, which are used to control force independently in the normal direction and close to the edge of friction cones, respectively. The recursive update rule of the discrete linearly constrained gradient flow can be also given:

$$\text{vec}(\boldsymbol{P}_{k+1}) = \text{vec}(\boldsymbol{P}_k) + \frac{\theta(\boldsymbol{I} - \boldsymbol{A}^+ \boldsymbol{A}) \text{vec}(\boldsymbol{P}^{-1} \boldsymbol{W}_i \boldsymbol{P}^{-1} - \boldsymbol{W}_p)}{\max_i(\left|(\boldsymbol{I} - \boldsymbol{A}^+ \boldsymbol{A}) \text{vec}(\boldsymbol{P}^{-1} \boldsymbol{W}_i \boldsymbol{P}^{-1} - \boldsymbol{W}_p)\right|)} \tag{22}$$

Where step length θ is introduced to ensure down convergence with a suitable value. $\max_i()$ represents absolute value of the maximum value of all the elements at the k-th iteration by Eq.(22).

This method is efficient but requires a valid initial point to start the gradient flow algorithm. The above mentioned safe initial values of grasping forces are as a valid initial point in Section 3. The final optimized result $\text{vec}(\boldsymbol{P}_\infty) \in \mathrm{R}^{(4k)^2}$ with exponentially convergence to the only the minimum value can be obtained by the iteration of Eq.(22). It is an important link that the safe initial value of grasping forces can be rearranged into a valid initial point $\text{vec}(\boldsymbol{P}_0) \in \mathrm{R}^{(4k)^2}$ by Eq.(23). The mutual formulas are described as follow:

$$\boldsymbol{f}_0 \in \mathrm{R}^{4k} \to \text{vec}(\boldsymbol{P}_0) \in \mathrm{R}^{(4k)^2} \tag{23}$$

$$\text{vec}(\boldsymbol{P}_\infty) \in \mathrm{R}^{(4k)^2} \to \boldsymbol{f}_\infty \in \mathrm{R}^{4k} \tag{24}$$

Where the iteration termination point of the linear gradient flow optimization algorithm can be also rearranged into a more compact grasping forces vector $\boldsymbol{f}_\infty \in \mathrm{R}^{4k}$ by Eq.(24).

5 Numerical examples of grasping forces optimization

The example corresponding to the case of four soft fingers (black spots) grasping a sphere object is shown in Fig.1. Assume that the radius of a sphere is 50 mm, and the weight is 5 N. The sphere center is the origin of the object reference frame, and the friction coefficients at contact points are $u_i = 0.5$, $u_{t,i} = 0.3$. All the codes are written in the Matlab (version 7.04). It is implemented with a desktop computer which has the 2.93 GHz CPU, 3.25 GB of memory. Four finger position vectors are given as follows:

$$\boldsymbol{r}_1 = [-35.3553, 0, 35.3553]^{\mathrm{T}},$$
$$\boldsymbol{r}_2 = [0, 50, 0]^{\mathrm{T}}, \quad \boldsymbol{r}_3 = [-25.0, -43.3013, 0]^{\mathrm{T}},$$
$$\boldsymbol{r}_4 = [30.6186, 17.6777, 35.3553]^{\mathrm{T}}$$

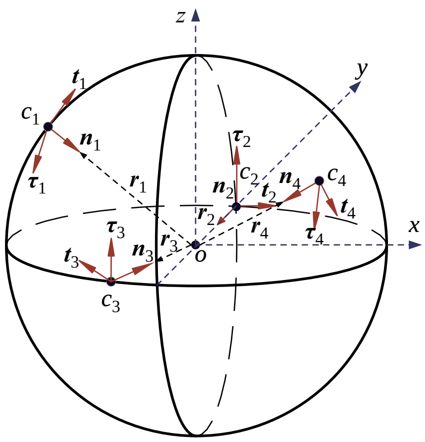


Fig.1 Soft multi-fingered hand grasping of a sphere object

It is easy to determine the inner normal vector and two orthogonal tangent vectors of all the contact points on the sphere. The unit vectors are calculated separately as follows:

$$\boldsymbol{n}_1 = [0.7071, 0, -0.7071]^{\mathrm{T}},\ \boldsymbol{t}_1 = [0.7071, 0, 0.7071]^{\mathrm{T}},$$
$$\boldsymbol{\tau}_1 = [0, -1, 0]^{\mathrm{T}};\ \boldsymbol{n}_2 = [0, -1, 0]^{\mathrm{T}},\ \boldsymbol{t}_2 = [1, 0, 0]^{\mathrm{T}},$$
$$\boldsymbol{\tau}_2 = [0, 0, 1]^{\mathrm{T}};\ \boldsymbol{n}_3 = [0.5, 0.866, 0]^{\mathrm{T}},$$
$$\boldsymbol{t}_3 = [-0.866, 0.5, 0]^{\mathrm{T}},\ \boldsymbol{\tau}_3 = [0, 0, 1]^{\mathrm{T}};$$
$$\boldsymbol{n}_4 = [-0.6124, -0.3536, -0.7071]^{\mathrm{T}},$$
$$\boldsymbol{t}_4 = [0.6124, 0.3536, -0.7071]^{\mathrm{T}}, \boldsymbol{\tau}_4 = [0.5, -0.866, 0]^{\mathrm{T}}$$

5.1 Force optimization under the effect of gravity

The weight coefficients $\varepsilon = 1.1$ and $\delta_0 = 0.02$ in initial step length $\boldsymbol{\delta}(0)$ remain unchanged in the simulation. The limitation conditions of safe initial values in algorithm are set as $\rho_i \geqslant 0$, $f_{ci}^3 > 0$ and $\left|f_{ci}^4\right| \leqslant 0.3 f_{ci}^3$ for the i-th finger in each loop. Assume that a gravity wrench acting on the object is $\boldsymbol{f}_{\mathrm{e}} = [0, 0, -5, 0, 0, 0]^{\mathrm{T}}$.

The computation time is 0.0336 s after 65 times of iterations in the program by Eq.(14)~(16). The valid initial value of grasping forces is obtained with a margin of more than 0:

$$\boldsymbol{f}_0 = [1.8817, -1.3927, 5.4983, -0.0063, -1.1667, 4.3946, 8.7914, 0.0143, 1.6005, 4.949, 10.082, -0.0173, -4.2103, -2.00636.7365, 0.0117]^{\mathrm{T}}$$

The valid initial value $\boldsymbol{f}_0$ is converted to a valid initial point of the linearly constrained gradient flow algorithm by Eq.(23). Here, choosing the two weight matrices of the cost index $\boldsymbol{W}_p = \boldsymbol{I} \in \mathrm{R}^{16\times16}$ and $\boldsymbol{W}_i = 10^{-3}\boldsymbol{I} \in \mathrm{R}^{16\times16}$ and iteration step length $\theta = 0.05$, guarantee that these parameters are the same in the following simulation. The detection condition during the optimization of the gradient flow algorithm is set as $\rho_i > 0$ and $f_{ci}^3 > 0$.

The final optimized value of grasping forces after 300 times of iterations is obtained based on the valid initial values with a margin of more than 0.

$$\boldsymbol{f}_\infty = [0.6576, -0.5901, 1.2751, -0.0033, 0.0017, 2.8486, 4.7069, 0.0238, 0.3022, 3.4245, 5.9354, -0.0282, -2.3188, -1.0200, 3.5017, 0.0107]^{\mathrm{T}}$$

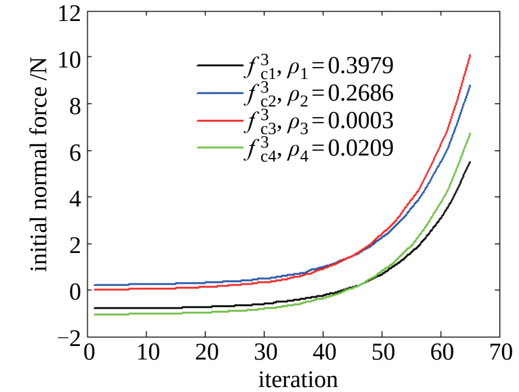
Normal grasping forces of the valid initial values with a margin of more than 0 in iteration are shown in Fig.2(a). Once the safety margins are detected to be greater than 0 at the same time, the algorithm will be automatically terminated and the safe initial values are obtained. Clearly, Fig.2(b) shows the optimized normal grasping forces and the final margins based on the gradient flow algorithm. Fig.2(c) shows the change of safety margins of the optimized values in iteration. It is found that the safety margin $\rho_4 < 0$ from the 94-th to 300-th loop iterations, which indicates that the finger will lose contact at the contact point in Fig.2(c). Even if a valid initial value is provided, it still has a safety margin of less than 0 during the optimization of the gradient flow algorithm, indicating that the grasping forces at the contact point don't satisfy friction cone constraints. Increasing the safety margin of the initial values will greatly reduce the occurrence of such phenomena.

In the next simulation, the safety margin is increased to 20%. Implement the same algorithm to obtain the safe initial values of grasping forces with a margin of more than 20%:

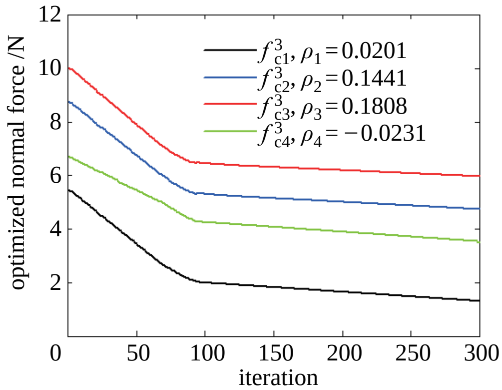
$$\boldsymbol{f}_0 = [3.2296, -2.5268, 11.4984, -0.0063, -2.5501, 7.6842, 16.9618, 0.0143, 3.3656, 8.2386, 19.6768, -0.0173, -6.9826, -3.6801, 14.1611, 0.0117]^{\mathrm{T}}$$

The final optimized values of grasping forces are obtained based on safe initial values with a safety margin of more than 20%:

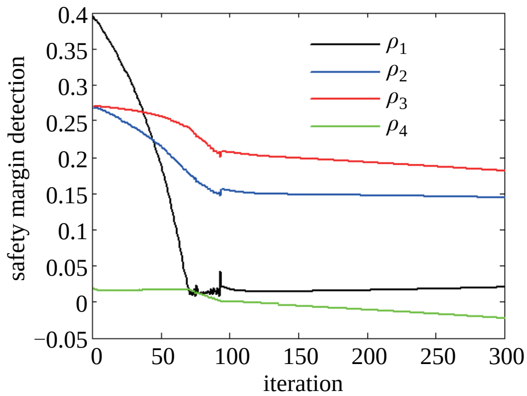
$$\boldsymbol{f}_\infty = [1.0690, -0.9872, 2.0839, -0.0031, -0.0306, 3.7720, 7.2506, 0.0277, 0.4383, 4.1793, 9.1604, -0.0336, -3.0074, -1.5640, 6.1661, 0.0103]^{\mathrm{T}}$$



(a) Normal forces of valid initial values with 0 margin



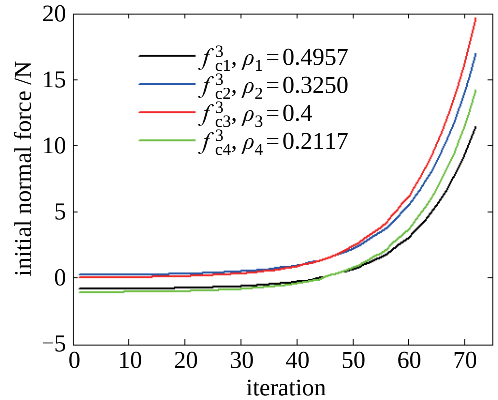
(b) Optimized normal forces based on the gradient flow algorithm



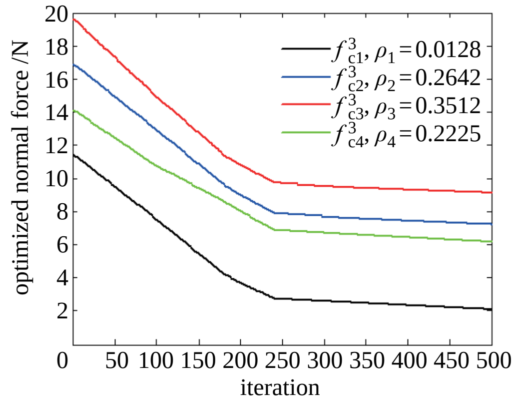
(c) The change of safety margins of the optimized values

Fig.2 Initial and optimized normal grasping forces and the safety margin changes with 0 margin in iteration

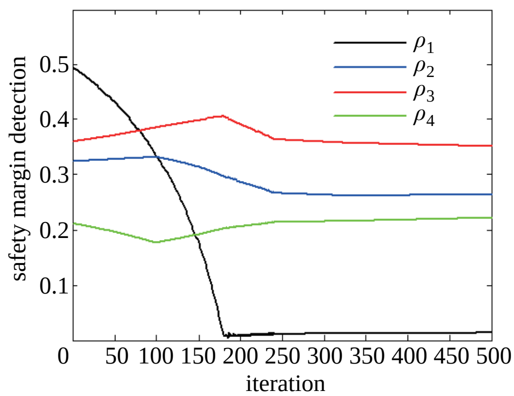
Fig.3(a) shows the change of normal grasping forces in safe initial grasping forces with a margin of more than 20%. Compared with Fig.3(a) and Fig.2(a), the initial values with 20% margin are significantly greater than those with 0 margin. Therefore, when the safety margin increases, the safe initial values will gradually increase, especially the normal grasping force components. Obviously in Fig.3(b), the final optimized values are greater than the optimized values in Fig.2(b). When the initial values are not the same, the optimization results are different, but the same initial values will correspond to the unique optimization result. Fig.3(c) shows the change of safety margins of the optimized values in iteration. In the optimization of the gradient flow algorithm, the safety margins at the contact points of four soft fingers are greater than 0, so this is a true valid optimization result.



(a) Normal forces of safe initial values with 20% margin



(b) Optimized normal forces based on the gradient flow algorithm



(c) The change of safety margins of the optimized values

Fig.3 Initial and optimized normal grasping forces and the safety margin changes with 20% margin in iteration

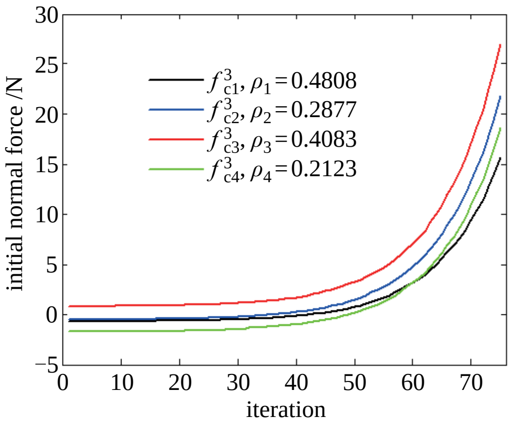
5.2 Force optimization under the external force wrench

With an external force wrench $\boldsymbol{f}_{\rm e} = [-1, -2.5, -6, 0.3, -0.2, 0.5]^{\rm T}$, and other parameters remaining unchanged, the safety margin of initial values is set as $\rho_i \geqslant 20\%$. The computation time of the initial values method is about 0.0387 s after 75 times of iterations, and the safe initial value of grasping forces with a margin of more than 20% is:

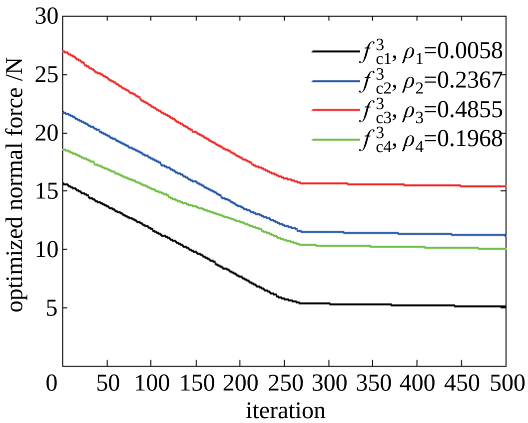
$$\boldsymbol{f}_0 = [4.3725, -3.7239, 15.6440, -0.0049, -3.3327, 10.4442, 21.7662, 0.018, 4.4708, 10.3838, 27.0187, -0.0177, -8.8669, -5.3194, 18.5655, 0.007]^{\mathrm{T}}$$

After the gradient flow algorithm optimization, the final optimized value of grasping forces with a margin of more than 0 is:

$$\boldsymbol{f}_\infty = [2.4727, -2.5148, 5.0157, -0.0048, -0.5765, 5.9848, 11.1398, 0.0380, 0.9506, 5.4779, 15.2816, -0.0392, -4.7746, -3.0298, 9.9570, 0.0078]^{\mathrm{T}}$$



(a) Safe initial grasping forces with safety margin



(b) Final optimized grasping forces with safety margin

Fig.4 Initial and optimized normal grasping forces in iteration

Fig.4(a) shows the corresponding trajectories of the normal grasping forces in safe initial grasping forces with safety margin of more than 20%. Fig.4(b) shows the convergence of normal grasping forces in final optimized grasping forces based on the gradient flow algorithm. It is very clear that we can get more realistic optimization results by increasing the safety margin.

5.3 Analysis

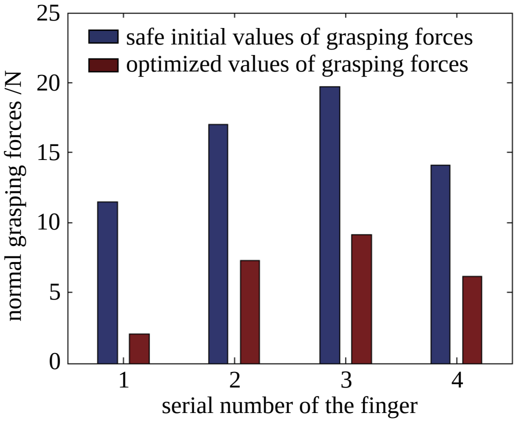
5.3.1 Limit conditions and influence factors

In order to obtain the safe initial values $\boldsymbol{f}_0$ from the iteration of Eq.(12)∼(14), the limit conditions must be satisfied in iteration for each finger by Eq.(1)∼(4) and Eq.(14)∼(16), that is $\rho_i \geqslant \lambda_{ci}/f_{ci}^3$, $f_{ci}^3 > 0$ and $\left|f_{ci}^4\right| \leqslant u_{t,i} f_{ci}^3$. The desired value of the safety margin can be set by Eq.(4). They are used as vector detectors to automatically exit the iteration of initial values in Eq.(12)∼(14). When the safety margin increases, the safe initial values and the corresponding safety margins will increase. If initial values can't satisfy these limit conditions, indicate that: 1) the denominator of safety margin is equal to 0; 2) the normal component of the grasping force is less than 0; 3) it doesn't satisfy the limit conditions. It is possible to re-adjust the value of the safety margin from small to large.

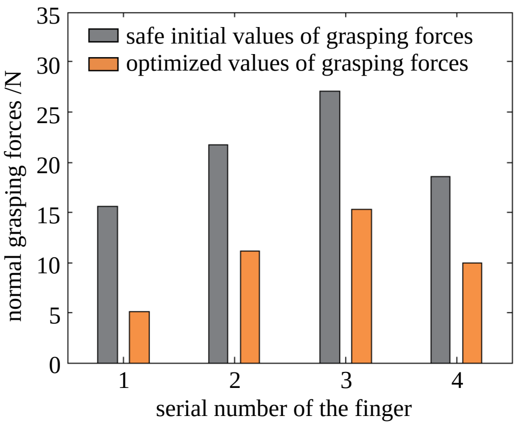
When the safe initial values are converted to a valid initial point of the linear gradient flow optimization algorithm, the external force wrench $\boldsymbol{f}_{\mathrm{e}}$, grasp matrix $\boldsymbol{G}$, friction coefficient u_i, torsional coefficient $u_{t,i}$ and step length θ will affect the final grasping force optimization result. When these conditions are fixed, the safety margin of initial values will also affect the optimized grasping forces and the corresponding safety margin. When the safety margin of initial values increases from 0 to 20%, it is clear by comparing Fig.2 and Fig.3 that the optimized safety margin of the grasping force of the first finger decreases from 0.0201 to 0.0128, and the optimized safety margin of the fourth finger increases from −0.0231 to 0.2225, and the safety margins of the other two fingers also increase accordingly. Therefore, increasing the safety margin of the initial values may not directly increase the optimized safety margin of the grasping forces, but can improve the optimization results of the gradient flow algorithm.

5.3.2 Comparison of grasping forces optimization

After eliminating the so-called unreliable initial values, this section focuses on the degree of grasping forces optimization based on safety margin detection. Here, the safe initial values with a margin of more than 20% and the optimized data are derived from Fig.3(a), (b) and Fig.4(a), (b). Compared with the initial values, the optimization degree of the normal grasping forces of the four fingers is [551.77%, 233.94%, 214.8%, 229.66%] and [311.9%, 195.39%, 176.81%, 186.46%]. Performance comparisons of normal grasping forces are shown through the histogram under the effect of gravity and the external force wrench in Fig.5. Clearly, although the safe initial values of grasping forces also satisfy friction cone constraints, compared with the final optimized values of grasping forces, especially the normal grasping forces, the safe initial values are too conservative and aren't optimal as indicated. As the normal grasping forces turns smaller after optimization, the tangential vectors and torsion components are optimized naturally. It can be seen that the safe initial values with a margin will generate the optimized values of grasping forces based on the gradient flow optimization algorithm.



(a) Comparison of normal forces under gravity



(b) Comparison of normal forces under external force wrench

Fig.5 Comparison of grasping forces optimization

In general, the choice of the initial values will greatly influences the final result of the gradient flow algorithm. Even with the valid initial values, it can't guarantee that the finger contact points always satisfy the friction cone constraints during the optimization of the gradient flow algorithm. When the safety margin increases, the initial values and the final optimization result will be larger, but the same initial values will correspond to the unique optimization result. Although the simulation is focused on soft finger grasping, the algorithm can be applied to hard finger grasping with friction because they have the same principles and detection methods.

6 Conclusion

This paper mainly studies the optimization problem of grasping forces based on safety margin detection. In order to more intuitively reflect the margin of the grasping force at contact points, a safety margin is defined, and an iteration method of safe initial values based on the desired safety margin detection is presented. The iterative formula of the initial values of the grasping forces is derived directly from the force closure formula. The obtained safe initial values are converted to an initial point of the classical linearly constrained gradient flow optimization algorithm. It is found that even with the valid initial values, it can't guarantee that the initial values always satisfy the friction cone constraints during the optimization. In such a case, safe initial values obtained by appropriately increasing the safety margin are effective to provide a remedy. By using this detection method, the initial values that don't satisfy the requirements can be excluded during the gradient flow optimization process. The choice of the initial values will greatly influence the final result of the gradient flow algorithm. Different safe initial values will result in different optimization results, but the same initial values will correspond to the unique optimization result. Finally, numerical examples of grasping forces optimization show the effectiveness of the optimization method based on safety margin detection.

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